3D Numerical Evaluation of HTSC Levitation Forces Using a Novel Technique Based on the Control Volume Method

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Abstract —**A novel three-dimensional numerical modeling technique for solving problems involving high temperature superconducting materials is presented. This technique is implemented in the control volume method using the magnetic vector potential and the electric scalar potential formulations ($\boldsymbol{A}$-j). This technique is based on the evaluation of surface integrals instead of volume ones for the divergence terms. A power-law relationship is used to model the nonlinear conductivity characteristic of superconductors. The simulation results are compared with experiments done by Foo and Moon [1,2] concerning the vertical and the lateral levitation forces created between a permanent magnet and a high temperature superconducting bulk.**

I. INTRODUCTION

THE superconducting levitation is based on the interaction between a permanent magnet (PM) and a high temperature-superconductor (HTSC). Due to its unique characteristics, it has demonstrated tremendous potential for several applications such as the magnetic levitation, the noncontact transport, and the flywheel energy storage, etc. In connection with these applications, it is very important to calculate precisely the levitation forces between the PM and the HTSC bulk. Several works of 3D modeling are were proposed for the calculation of these forces in vertical and lateral cases, where the finite elements method (FEM) [3] and the control volume method (CVM) [4] are generally used for the resolution of the equations of the treated physical phenomena. In our previous works [4], we have used the CVM for a structured grid and we have proposed a new method for treating correctly the rotational volume terms of electromagnetic equations which obeys to a conservation rotational law. The major problem of the CVM is related to its structured grid which does not enable us to model devices having complex geometries. For this reason, we propose in this paper a novel technique which can be used for unstructured grid and do not need any special treatment of rotational volume terms. This version of CVM consists in using the control volumes grid build by connecting the gravity centers of nearby elements (Fig.1) of an initial mesh made of unstructured triangular prisms. This technique can also be applied to structured grids. In this technique, we evaluate the surface integrals instead of the volume integrals of divergence terms. A power-law relationship is used to model the nonlinear conductivity characteristic of superconductors.

II. BASIC FORMULATION

When a PM moves above a superconductor, the magnetic field of the PM penetrates the superconductor creating shielding currents or fluxoids in it. In this case, magnetic forces are produced by the interaction between the shielding current and the applied magnetic field. These forces can be evaluated by the Lorentz formula. For the calculation of the external magnetic field $\boldsymbol{B_m}$, a PM with a magnetic moment $\boldsymbol{m}$ has been modeled by succession of loops with surface current density $\boldsymbol{J_{PM}} = \boldsymbol{m}$Ù$\boldsymbol{n}$ [1]. By using the Biot-Savart law, $\boldsymbol{B_m}$ is calculated in the whole domain. To evaluate the current density $\boldsymbol{J}$ inside the HTSC bulk, the ($\boldsymbol{A}$-j) formulation with the Coulomb gauge is used:

$$\begin{aligned} &-\nabla\cdot(\nu_0\nabla)\boldsymbol{A}+\sigma(\boldsymbol{E})(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi)=\boldsymbol{J_s} \\ &\nabla\cdot\left\{-\boldsymbol{\sigma}(\boldsymbol{E})(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi)\right\}=0 \end{aligned} \quad . \qquad (2)$$

The relation between the electric field $\boldsymbol{E}$ and the current density $\boldsymbol{J}$ is non nonlinear. The effective conductivity of the superconductor $\sigma(\boldsymbol{E})$ can be expressed by a power-law [3]:

$$\sigma(\boldsymbol{E})=\frac{Jc}{Ec}\left(\frac{|\boldsymbol{E}|}{Ec}\right)^{\frac{1}{n}-1} . \qquad (3)$$

The parameters J_c and n are derived from the measured current-voltage characteristics of the sample.

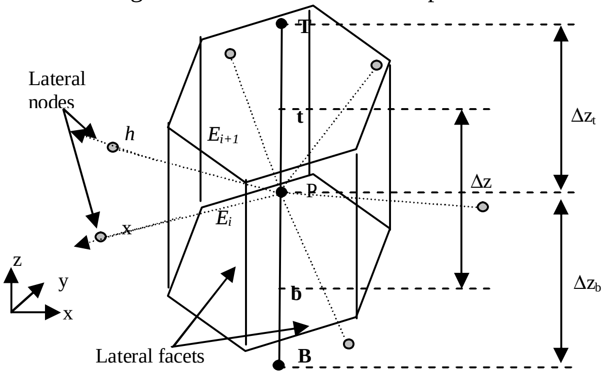

Fig. 1. Novel control volume scheme.

III. CONTROL VOLUME METHOD (CVM)

The CVM consists in dividing the domain into a number of subdomains or control volumes such as there is one control volume D_p surrounding each node P. The new scheme of the control volume D_p is displayed in Figure 1. D_p is limited by several facets related to neighboring nodes of P. So the system of equations (2) will be integrated over the control volume D_p as follows:

$$\begin{aligned} &-\iiint_{D_P}\nabla\cdot(\nu_0\nabla)\boldsymbol{A}\,d\tau+\iiint_{D_P}\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t}+\nabla V\right)d\tau=\iiint_{D_P}\boldsymbol{J}_s d\tau \\ &\iiint_{D_P}\nabla\cdot\left[-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t}+\nabla V\right)\right]d\tau=0 \end{aligned} \quad . \qquad (4)$$

By using the theorem of Ostrogradski, the divergence volume integrals of (4) are transformed into surface integrals. For example, for the Ax component, one obtains:

$$\iiint_{D_P}\nabla.(\nu_0\nabla Ax)d\tau=\iint_{\sum dsl_{ai}}\nu_0\nabla Axds_{l_{ai}}+\iint_{\substack{Sk\\k=T;B}}\nu_0\nabla Axds_T \;. \qquad (5)$$

The right hand side terms in equation (5) represent the flux of the gradient magnetic potential Ax through the lateral facets and the top and bottom facets. To calculate top and bottom terms, first order approximation is used. For example, the flux in the top facet is:

$$\iint\nu\nabla A_x ds\Big|_T=\nu_t\frac{A_x^T-A_x^P}{\Delta z_t}\Delta s_T \;. \qquad (6)$$

To calculate the flux in the lateral facets, the derivative of the magnetic potential Ax is expressed in referential (R') defined by (ξ,h) coordinates. The passage from the referential (R') to the referential (R) defined by (x,y) coordinates is obtained by the Jacobian matrix. So, the flux in the lateral facets can be expressed as:

$$\iint_{\sum dsl_{ai}}\nu\nabla Axds_{l_{ai}}=\sum_{\substack{i=1..nEi\\R=\eta,\zeta}}a_{REi}Ax_{REi}-apAxp \qquad (7)$$

where $a_{REi}=(ax_{REI}+ay_{REi})$ and $ap=\sum_{R=\eta,\zeta}a_{REi}$.

Therefore, the obtained coefficients a_{kEi} describe the physical and geometrical properties of the control volume or cell surrounding each node. All integral terms resulting from system (4) are evaluated. An algebraic system of equations is constructed with appropriate boundary conditions and is expressed as:

$$\begin{bmatrix}[M_1] & [0]\\ [M_2] & [0]\end{bmatrix}\begin{bmatrix}\dot{A}\\ \dot{V}\end{bmatrix}+\begin{bmatrix}[N_{11}] & [N_{12}]\\ 0 & [N_{22}]\end{bmatrix}\begin{bmatrix}A\\ V\end{bmatrix}=\begin{bmatrix}J_S\\ 0\end{bmatrix}. \qquad (8)$$

The column matrix $[A;V]$ denotes the nodal values of the magnetic vector and electric scalar potentials and $[\dot{A};\dot{V}]$ their time derivatives. $[J_s;0]$ represents the source column vector of the PM excitation. The global CVM matrices *[M]* and *[N]* are non symmetric and nonlinear according to the definition of conductivity given by (3). At each step, the Crank-Nicolson method is employed to integrate the algebraic system (8) which is solved iteratively until convergence is reached.

IV. RESULTS

The developed models and simulation programs are used to evaluate the vertical and lateral forces acting on the PM described in the experiments done by Foo and Moon [1,2] as shown in Figures 2 and 3, for two cooling conditions ; with field cooling (FC) and zero field cooling (ZFC). Good agreements for both lateral and vertical forces calculation are obtained compared to experimental data. The absolute maximum errors are 0.79 N for the vertical displacement and 0.19 N for the lateral displacement.

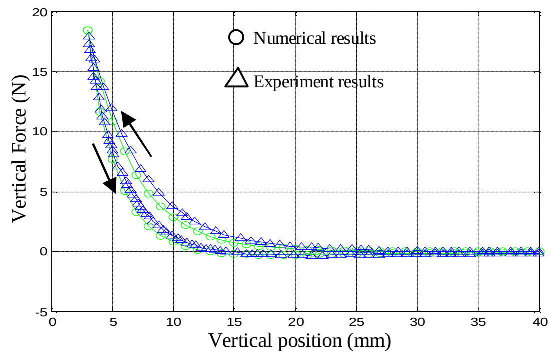

Fig. 2. Vertical force as a function of vertical displacement at ZFC condition.

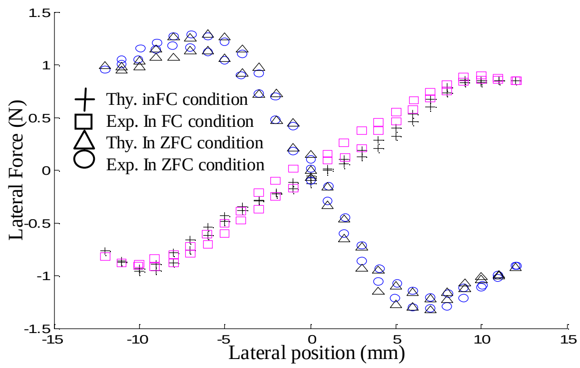

Fig. 3. Lateral force as a function of lateral displacement at ZFC and at FC conditions

V. REFERENCES

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